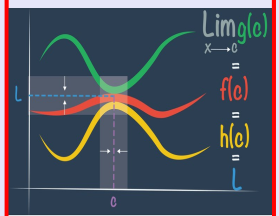
Calculus I

Lecture 47



Feb 19-8:47 AM

Formulas from Pre Calc or College Algebra

$$\boxed{\sum_{i=1}^{n} c = cn}$$

$$\boxed{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$$

$$\boxed{\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}}$$

$$\boxed{\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2}$$

$$\sum_{i=1}^{n}(a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$$

May 8-9:52 AM

Suppose $f(x) \geq 0$ and Continuous for all values from a to b.

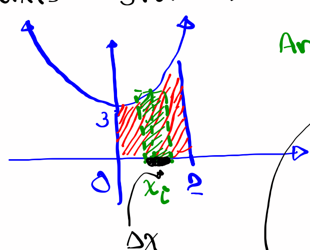
The area below $f(x)$, above x-axis from $x=a$ to $x=b$ is given by

$$\boxed{A = \lim_{n \to \infty} \sum_{i=1}^{n} \Delta x \cdot f(x_i)}$$

where $\Delta x = \frac{b-a}{n}$, $x_i = a + i\Delta x$

May 9-8:50 AM

Find the area below $f(x) = x^2 + 3$, above the x-axis from $x=0$ to $x=2$.



Area of Such rectangle

$\Delta x \cdot f(x_i) = A_i$

$a=0$, $b=2$

$\Delta x = \frac{b-a}{n} = \frac{2}{n}$

$x_i = a + i \cdot \Delta x = \frac{2i}{n}$

$= \frac{2}{n} \cdot f\left(\frac{2i}{n}\right)$

$= \frac{2}{n}\left[\left(\frac{2i}{n}\right)^2 + 3\right]$

$= \frac{2}{n}\left[\frac{4i^2}{n^2} + 3\right]$

$= \frac{8i^2}{n^3} + \frac{6}{n}$

$A = \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{8i^2}{n^3} + \frac{6}{n}\right)$

$= \lim_{n \to \infty} \left[\sum_{i=1}^{n} \frac{8i^2}{n^3} + \sum_{i=1}^{n} \frac{6}{n}\right]$

$= \lim_{n \to \infty} \left[\frac{8}{n^3} \cdot \sum_{i=1}^{n} i^2 + \frac{1}{n} \sum_{i=1}^{n} 6\right]$

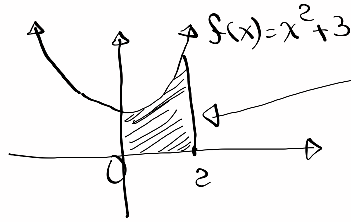
May 9-8:55 AM

$$A = \lim_{n\to\infty}\left[\frac{8}{n^3}\cdot\sum_{i=1}^{n} i^2 + \frac{1}{n}\sum_{i=1}^{n} 6\right]$$

$$= \lim_{n\to\infty}\left[\frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{1}{n}\cdot 6n\right]$$

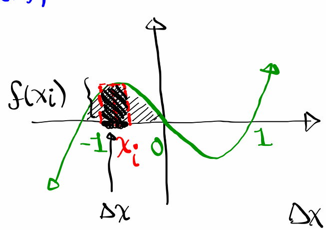
$$= \lim_{n\to\infty}\left[\frac{16n^3 + \cdots}{6n^3} + 6\right]$$

$$= \frac{16}{6} + 6 = \frac{8}{3} + 6 = \frac{8}{3} + \frac{18}{3} = \boxed{\frac{26}{3}}$$



May 9-9:04 AM

Consider $f(x) = x^3 - x$, find the area below $f(x)$, above x-axis, from $x=-1$ to $x=0$.



$$A = \lim_{n\to\infty}\sum_{i=1}^{n}\Delta x\cdot f(x_i)$$

$$\Delta x = \frac{b-a}{n}, \quad x_i = a + i\Delta x$$

$$\Delta x = \frac{0-(-1)}{n} = \frac{1}{n}$$

$$x_i = -1 + i\cdot\frac{1}{n} = -1 + \frac{i}{n}$$

$$A = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left[x_i^3 - x_i\right] \qquad (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left[\left(-1+\frac{i}{n}\right)^3 - \left(-1+\frac{i}{n}\right)\right]$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left[-1 + \frac{3i}{n} - \frac{3i^2}{n^2} + \frac{i^3}{n^3} + 1 - \frac{i}{n}\right]$$

May 9-9:08 AM

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left[\cancel{-1} + \frac{3i}{n} - \frac{3i^2}{n^2} + \frac{i^3}{n^3} \cancel{+1} - \frac{i}{n}\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left[\frac{2i}{n} - \frac{3i^2}{n^2} + \frac{i^3}{n^3}\right]$$

$$= \lim_{n\to\infty} \left[\frac{2}{n^2}\cdot\sum_{i=1}^{n} i - \frac{3}{n^3}\sum_{i=1}^{n} i^2 + \frac{1}{n^4}\sum_{i=1}^{n} i^3\right]$$

$$= \lim_{n\to\infty} \left[\frac{2}{n^2}\cdot\frac{n(n+1)}{2} - \frac{3}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{1}{n^4}\cdot\left[\frac{n(n+1)}{2}\right]^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{2n^2+\cdots}{2n^2} \to 1 - \frac{6n^3+\cdots}{6n^3} \to 1 + \frac{n^4+\cdots}{4n^4} \to \frac{1}{4}\right]$$

$$= 1 - 1 + \frac{1}{4} = \boxed{\frac{1}{4}} \checkmark$$

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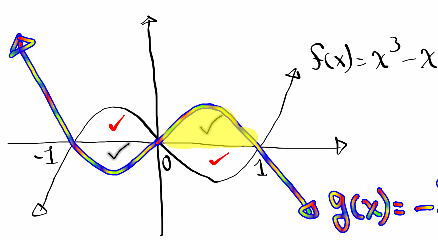

$$g(x) = -f(x) = -(x^3 - x) = \boxed{x - x^3}$$

$a = 0 \qquad b = 1$

$\Delta x = \frac{b-a}{n} = \frac{1}{n}, \quad x_i = a + i\Delta x = \frac{i}{n}$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(x_i - x_i^3\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{i}{n} - \frac{i^3}{n^3}\right)$$

$$= \lim_{n\to\infty} \left[\frac{1}{n^2}\cdot\sum_{i=1}^{n} i - \frac{1}{n^4}\sum_{i=1}^{n} i^3\right]$$

$$= \lim_{n\to\infty} \left[\frac{1}{n^2}\cdot\frac{n(n+1)}{2} - \frac{1}{n^4}\cdot\left[\frac{n(n+1)}{2}\right]^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{n^2+\cdots}{2n^2} \to \frac{1}{2} - \frac{n^4+\cdots}{4n^4} \to \frac{1}{4}\right] = \frac{1}{2} - \frac{1}{4} = \boxed{\frac{1}{4}} \checkmark$$

May 9-9:26 AM

Anti derivative & Integration

$f'(x) = ? \rightarrow f(x)$

$\int f'(x)\,dx = f(x) + C$

integral

with respect to x

$f'(x) = 2x \rightarrow f(x) = x^2 + C$

$\int 2x\,dx = x^2 + C$

$\int \cos x\,dx = \sin x + C$

$\int (\sec^2 x + 2x)\,dx = \tan x + x^2 + C$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad , n \neq -1$

$\int (x^3 - 4x)\,dx = \frac{x^{3+1}}{3+1} - \frac{4x^{1+1}}{1+1} + C$

$= \boxed{\frac{1}{4}x^4 - 2x^2 + C}$

May 9-9:36 AM

Find $\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{2}{3}\sqrt{x^3} + C$

$= \frac{2x\sqrt{x}}{3} + C$

Evaluate the answer at 4 and at 1.

Find $\boxed{\text{the difference.}}$

At 4 $\rightarrow \frac{2 \cdot 4 \cdot \sqrt{4}}{3} + C = \frac{16}{3} + C$

At 1 $\rightarrow \frac{2 \cdot 1 \cdot \sqrt{1}}{3} + C = \frac{2}{3} + C$

$\frac{16}{3} + C - \left(\frac{2}{3} + C\right) = \boxed{\frac{14}{3}}$ *

May 9-9:44 AM

Suppose $f(x) = \int x^3\,dx = \frac{x^4}{4} + C$

Find $f(1) - f(0) = \left(\frac{1^4}{4} + C\right) - \left(\frac{0^4}{4} + C\right)$

$= \frac{1}{4} + C - C = \boxed{\frac{1}{4}}$

Suppose $f(x) = \int (x^2+3)\,dx = \frac{x^3}{3} + 3x + C$

Find $f(2) - f(0) = \left[\frac{2^3}{3} + 3(2) + C\right] - \left[\frac{0^3}{3} + 3(0) + C\right]$

$= \frac{8}{3} + 6 + C - C$

$= \frac{8}{3} + 6 = \frac{8}{3} + \frac{18}{3} = \boxed{\frac{26}{3}}$

May 9-9:48 AM